11º CONGRESSO NACIONAL DE BIOMECÂNICA
A. P. G. Castro, C. Pina, P. R. Fernandes, C. Santos, J. Folgado & A. Amaro (Eds)
Sesimbra, Portugal, 21-22 de Fevereiro, 2025

# FULLY CARTESIAN COORDINATES WITH A GENERIC RIGID BODY: AN APPLICATION TO BIOMECHANICS

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**KEYWORDS:** Multibody Dynamics, Fully Cartesian Coordinates, Generic Rigid Body, Motion Analysis, Biomechanics

## 1 INTRODUCTION

Multibody dynamic-based methodologies have been widely and successfully used for the biomechanical analysis of human motion, offering an efficient and straightforward approach to modeling complex mechanical systems. Various multibody formulations have been proposed for studying (bio)mechanical systems, differing in how bodies are modeled, the nature of the coordinates used, and the complexity of constraint and state equations. In a series of works [1,3], Gonçalves et al. extended the planar fully Cartesian coordinates with a Generic Rigid Body (FCC-GRB) multibody formulation [4] to analyze spatial systems, validating its effectiveness through a series of benchmark problems.

The present work explores the application of the spatial FCC-GRB formulation to the analysis and simulation of biomechanical systems, highlighting its advantages and limitations compared to other global formulations. It outlines the general principles for modeling biomechanical systems, including musculotendon units (MTU) and contact models. Additionally, this study addresses the use of mixed coordinates for conducting inverse kinematic analyses using experimental data obtained from traditional optoelectronic marker-based system. To evaluate the performance of the FCC-GRB formulation, it was applied to the analysis of different biomechanical models, ranging from single-joint to full-body models.

## 2 FCC-GRB FORMULATION

The FCC-GRB is a multibody formulation designed to preserve the main characteristics of the two most common global formulations: the Cartesian coordinates formulation (CCF) and the natural coordinates formulation (NCF). Similar to the CCF, the rigid bodies are modeled with a fixed and predetermined kinematic structure, simplifying the modeling procedure and generating diagonal mass matrices with a high physical meaning. However, like the NCF, the rigid bodies are algebraically defined using only Cartesian coordinates of points and vectors. Particularly, in spatial systems, each rigid body is defined with one reference point located at its center of mass and three unit vectors (see Fig. 1a). This approach has a deep impact in the straightforwardness and efficiency of the formulation, as it reduces the degree of nonlinearity of the kinematic constraints. Specifically, the kinematic constraint equations for the FCC-GRB exhibit at most a quadratic dependency on the generalized coordinates [1]. Moreover, the kinematics of any point or vector in the system can be determined through a set of constant transformation matrices ($\mathbf{C}^P$ and $\mathbf{C}^{\mathbf{v}}$), simplifying the definition of kinematic pairs, the application of external forces and moments, and determination of the internal forces of the system [1,4].

## 3 FCC-GRB IN BIOMECHANICS

The inherent characteristics of the FCC-GRB formulation make it particularly well-suited for biomechanical applications. Specifically, the elements used to define the rigid body have a direct correspondence with the inertial data typically found in anthropometric databases. The rigid body vectors can be aligned with anatomical joint axes, allowing for a direct representation of rotational directions. Describing the system's kinematics through the constant transformation matrices **C** not only simplifies the skeletal system definition but also facilitates the modeling of MTU kinematics and its functional behavior. For example, in the case of a generic MTU composed of $n_m$ segments (see Fig. 1a), its total length ($l^{MT}$) can be directly calculated from the generalized coordinates as

$$l^{MT} = \sum_{n=1}^{n_m} \sqrt{\left(\mathbf{C}_j^{P_n}\mathbf{q}_j - \mathbf{C}_i^{P_{n-1}}\mathbf{q}_i\right)^T \left(\mathbf{C}_j^{P_n}\mathbf{q}_j - \mathbf{C}_i^{P_{n-1}}\mathbf{q}_i\right)} \tag{1}$$

where $\mathbf{C}_j^{P_n}$ and $\mathbf{C}_i^{P_{n-1}}$ represent the transformation matrices of the points $P_n$ and $P_{n-1}$, which define the muscle segment *n*, and $\mathbf{q}_i$ is the vector of the generalized coordinates of the *i-th* rigid body. This characteristic is also applicable to the modeling of contact/impact problems. The equivalent generalized force for a given contact force pair can be easily computed using the transformation matrix of the contact point ($\mathbf{C}^{P_C}$) and the corresponding contact force ($\mathbf{f}_C$), as

$$\mathbf{g}_{ij}^{\mathbf{f}_C} = \left\{ \left(-\mathbf{C}_i^{P_C}\mathbf{f}_C\right)^{\mathrm{T}} \quad \left(\mathbf{C}_j^{P_C}\mathbf{f}_C\right)^{\mathrm{T}} \right\}^{\mathrm{T}} \tag{2}$$

## 4 RESULTS AND DISCUSSION

Figures 1b and c present the results from an inverse dynamic analysis of gait, demonstrating good agreement with values reported in the literature. The FCC-GRB formulation proved to be an efficient and straightforward methodology for both modeling and analyzing biomechanical systems, enabling the implementation of new models with minimal need for extensive knowledge in computational mechanics.

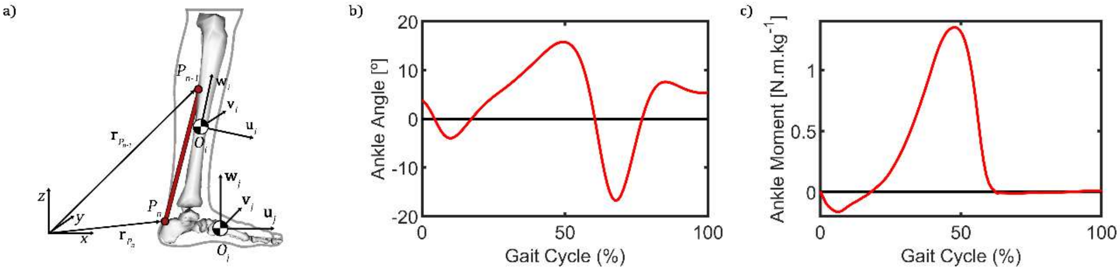


Figure 1 – a) Representation of a generic rigid body using the FCC-GRB Formulation with one MTU; b) Ankle joint angular displacement during gait; c) Moment of force at the ankle joint during gait.

## ACKNOWLEDGEMENTS

The authors acknowledge Portuguese Army, through CINAMIL, and Fundação para a Ciência e a Tecnologia (FCT) for their financial support via the projects ELITE III - Enhancement LITe Exoskeleton, LAETA Base Funding (DOI: 10.54499/UIDB/50022/2020) and LAETA Programmatic Funding (DOI: 10.54499/UIDP/50022/2020).

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